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On the Worldsheet Formulation of the Six-Dimensional Self-Dual String

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Abstract

Despite recent evidence indicating the existence of a new kind of self-dual six-dimensional superstring, no satisfactory worldsheet formulation of such a string has been proposed. In this note we point out that a theory built from $\mathbf{Z}_4$ parafermions may have the right properties to describe the light-cone conformal field theory of this string. This indicates a possible worldsheet formulation of this theory.

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1 Introduction

Over the past year, evidence [1, 2, 3, 4] has accumulated for the existence of a new kind of closed superstring which

- has six-dimensional Lorentz invariance;
- is $N = (2, 0)$ spacetime supersymmetric; and
- whose massless modes are in a tensor supermultiplet.

This last property implies that this string has no graviton in its spectrum, and that it couples to an antisymmetric two-form $B_{\mu\nu}$ with self-dual field strength. This string, which is the first example of non-trivial infrared dynamics of a quantum theory in six flat dimensions, arises in certain compactifications of Type IIB strings or of M-theory when parallel five-branes coincide. There are also self-dual strings with $N = (1, 0)$ supersymmetry in six dimensions [5, 6, 7, 8], as well as $N = 2$ supersymmetric strings in four dimensions [9, 10].

The above properties, which concern only the macroscopic behavior of the self-dual string, can be summarized by noting [11] that the massless states of the self-dual string are the same as those of the classical six-dimensional Green-Schwarz superstring. (Recall that the six-dimensional Green-Schwarz string is indeed consistent at the classical level.) A microscopic description of the self-dual string, on the other hand, is lacking.

In this note we will *assume* that there exists a microscopic worldsheet formulation of the self-dual string, and seek to construct it. We should emphasize at the outset, however, that there is very little evidence to support the existence of such a microscopic formulation. Indeed, given that the self-dual string is necessarily strongly coupled (by virtue of its self-duality), it is hard to guess the properties of a microscopic description. In Ref. [12], for example, it is argued that a worldsheet formulation of the strongly-coupled string might exist because microscopic string states in six flat dimensions do not couple to the $B_{\mu\nu}$ field at zero momentum. However, such states are derivatively coupled, which would seem to imply only that the effective six-dimensional theory of the massless tensor multiplet is weakly coupled.

One clue to such a microscopic description comes from studies of black hole entropy, from which it is argued [11] that the self-dual string

- has an asymptotic degeneracy of BPS-saturated states consistent with a light-cone gauge worldsheet theory with central charge $c = 6$.

Indeed, in Ref. [11], the use of a weakly-coupled description of this string was justified by noting that the counting of BPS states should not change as the theory is deformed to weak coupling. This then gives rise to a worldsheet formulation of the self-dual string, which the authors of Ref. [11] took to be that of the six-dimensional

Green-Schwarz string. This theory is described by a light-cone gauge conformal field theory (CFT) consisting of four transverse bosons X^i and four worldsheet fermions S^a carrying spacetime spinor indices in the $2(\mathbf{2},\mathbf{1})$ representation of the $SU(2)\times SU(2)$ massless little group. It is well-known, however, that this theory is not Lorentz invariant. Thus, perhaps, this worldsheet formulation describes the self-dual string in some background in which six-dimensional Lorentz invariance is broken but in which the string is weakly coupled.

Indeed, the mere existence of the classical Green-Schwarz actions in six, four, and three dimensions might be taken as an additional piece of evidence of a microscopic worldsheet description of the self-dual string. In particular, the Green-Schwarz actions are "special" in the sense that they depend on the existence of a non-trivial κ-symmetry.

The purpose of this note is to give evidence for another, closely related worldsheet description of the self-dual string. The main guideline we follow is that the central charge of this worldsheet theory in light-cone gauge should be $c = 6$. Our procedure will be as follows. First, we will construct a partition function which has all of the properties necessary for a consistent interpretation as that of the self-dual string. Then, we will show how the formulation of this partition function naturally suggests a construction of the associated worldsheet conformal field theory.

2 Constructing the Partition Function

Let us begin by reviewing the partition function of the six-dimensional Green-Schwarz string. This partition function is given by

$$Z(\tau) \;=\; (\mathrm{Im}\,\tau)^{-2}\left|f_B(\tau) - f_F(\tau)\right|^2 \tag{2.1}$$

where the bosonic and fermionic contributions $f_B(\tau)$ and $f_F(\tau)$ are equal and given by:

$$f_B(\tau) \;=\; f_F(\tau) \;=\; 4\prod_{n=1}^{\infty}\left(\frac{1+q^n}{1-q^n}\right)^4 ~, \qquad q \equiv e^{2\pi i\tau} ~. \tag{2.2}$$

This infinite product can be written in terms of Jacobi ϑ_i-functions and the Dedekind η-function as

$$4\prod_{n=1}^{\infty}\left(\frac{1+q^n}{1-q^n}\right)^4 \;=\; \frac{{\vartheta_2}^2(\tau)}{\eta^6(\tau)} ~. \tag{2.3}$$

These functions are the characters of four worldsheet bosons and four Majorana-Weyl worldsheet fermions, which comprise the $c = 6$ light-cone gauge field content of the six-dimensional Green-Schwarz string.

The worldsheet formulation described in Ref. [11] then proceeds from this point, assuming such bosonic and fermionic fields on the worldsheet. However, as further

analyzed in Ref. [12], this approach leads to the well-known difficulties concerning Lorentz invariance. Indeed, the attempted solution to this problem proposed in Ref. [12] involves increasing the central charge beyond $c = 6$. By contrast, in this note we shall explore whether there exists an alternative formulation of the theory which does not involve simple worldsheet bosons and fermions, but which nevertheless retains the properties we desire.

In general, the problem we face is that we wish to realize or interpret the infinite product (2.2) in terms of the underlying characters of an unknown six-dimensional theory via an expression of the form $f = \eta^{-4} \sum \chi_1\chi_2\chi_3\chi_4$. Here η^{-4} is the character of four (transverse) free bosons, the summation is over the different sectors of the hypothetical theory, and χ_i are the characters of the worldsheet fields of that theory. Six-dimensional Lorentz invariance suggests that each term in the expression f should have precisely four such χ_i characters in a light-cone gauge formulation. Moreover, spacetime supersymmetry requires that there should exist two distinct ways of expressing the infinite product (2.2) in terms of the characters of that theory: one combination of characters f_B should correspond to the spacetime bosons, while the other set f_F should correspond to the spacetime fermions. For example, in the ten-dimensional Type II string, such inequivalent ways of writing the corresponding infinite product would be $f_B = {\vartheta_3}^4 - {\vartheta_4}^4$ and $f_F = {\vartheta_2}^4$; these have equivalent product representations as a result of the usual 'abstruse' Jacobi identity. By contrast, note that the *six*-dimensional Green-Schwarz string would require terms of the form $f_F = {\vartheta_2}^2$ and $f_B = ({\vartheta_3}^4 - {\vartheta_4}^4)^{1/2}$; the appearance of the undesired square root in this expression ultimately signals the breakdown of Lorentz invariance.

There exists an alternative realization of the infinite product (2.2) which has all of the desired properties [13, 14]. However, rather than relying on the characters of simple fermions, this realization makes use of the characters of the $\mathbb{Z}_4$ *parafermion* conformal field theory [15]. This $c = 1$ conformal field theory, which is equivalent [16] to that of a free boson on a circle of radius $\sqrt{3/2}$, can be realized as the coset $SU(2)_4/U(1)$, and has seven primary fields denoted $\{\phi^0_0, \phi^1_0, \phi^1_{\pm1}, \phi^2_0, \phi^2_{\pm1}\}$. Here each primary field ϕ^j_m has been labelled by its $SU(2)$ quantum numbers j and m. These primary fields are the parafermionic analogues of the different toroidal boundary conditions of an ordinary Majorana-Weyl fermion, and they generalize the identity, fermion, and spin-fields of the Ising model. The conformal dimension of each parafermionic primary field ϕ^j_m is given by $h^j_m = j(j+1)/6 - m^2/4$, and the corresponding character will be denoted $\chi^{2j}_{2m}(\tau)$. These characters, which satisfy the identity $\chi^{2j}_{2m} = \chi^{2j}_{-2m}$, are given by $\chi^{2j}_{2m} \equiv \eta c^{2j}_{2m}$ where $c^{2j}_{2m}(\tau)$ are the so-called parafermionic *string functions* [17]. As required, there exist two distinct linear combinations $A^{\rm boson}$ and $A^{\rm fermion}$ of quadrilinear products of these characters χ^{2j}_{2m} which reproduce the desired infinite product [14]:

$$4 \prod_{n=1}^{\infty} \left(\frac{1+q^n}{1-q^n} \right)^4 ~=~ A^{\rm boson} ~=~ A^{\rm fermion} ~, \tag{2.4}$$

where

$$\begin{aligned} A^{\rm boson} &= \eta^{-4}\left[4(\chi_2^2)^4 \;-\; 32(\chi_2^2)(\chi_2^4)^3\right] \\ A^{\rm fermion} &= \eta^{-4}\left[4(\chi_0^0+\chi_0^4)^3(\chi_0^2) \;-\; 4(\chi_0^2)^4\right] ~. \end{aligned} \tag{2.5}$$

Thus we are naturally led to a potential worldsheet formulation of the self-dual string for which the worldsheet theory consists not just of four transverse coordinate bosons, but also four $\mathbb{Z}_4$ parafermions. Each term in the expressions (2.5) would then correspond to a different bosonic or fermionic sector of this underlying theory.

Since each $\mathbb{Z}_4$ parafermion conformal field theory has central charge $c=1$, this proposal would seem to imply a total worldsheet light-cone central charge $c=8$, which disagrees with the desired value $c=6$. However, as guaranteed from the relations (2.4), this is ultimately not the case, for the minus signs within (2.5) imply a cancellation of states in which the original $c=8$ Fock space is "deformed" down to that of a $c=6$ theory [14, 18, 19]. This delicate cancellation is called an *internal projection*, and will be discussed below. Thus, we see that we have constructed a non-trivial realization of a $c=6$ worldsheet theory in terms of a larger parafermionic $c=8$ worldsheet theory, followed by an internal projection.

This is not all, however. In order to produce a fully consistent theory, the total partition function must be modular invariant. However, the naive partition function constructed from just the expressions in (2.5) is not modular invariant. Rather, for consistency, we find that we must add the additional contributions

$$\begin{aligned} B^{\rm boson} &= \eta^{-4}\left[8(\chi_0^0+\chi_0^4)^2(\chi_2^2)(\chi_2^4) \;-\; 4(\chi_0^2)^2(\chi_2^2)^2\right] \\ B^{\rm fermion} &= \eta^{-4}\left[4(\chi_0^2)^2(\chi_2^2)^2 \;-\; 16(\chi_0^0+\chi_0^4)(\chi_0^2)(\chi_2^4)^2\right] ~, \end{aligned} \tag{2.6}$$

so that the total modular-invariant partition function for this six-dimensional theory becomes

$$Z(\tau) ~=~ ({\rm Im}\,\tau)^{-2}\left\{|A^{\rm boson}-A^{\rm fermion}|^2 ~+~ 3\,|B^{\rm boson}-B^{\rm fermion}|^2\right\} ~. \tag{2.7}$$

On the face of it, the forced introduction of these extra B-sectors has the potential to destroy the two features of our partition function that we most wished to preserve: spacetime supersymmetry, and the central-charge reduction from $c=8$ to $c=6$. Remarkably, however, the B-sector expressions in (2.6) share the same properties as their A-sector counterparts [14, 18]. First, they actually preserve the spacetime supersymmetry, for $B^{\rm boson}$ and $B^{\rm fermion}$ are found to be equal as functions of τ. Second, these expressions also exhibit the same minus-sign cancellations that reduce the apparent central charge from $c=8$ to $c=6$. Indeed, in complete analogy to (2.4), this central-charge reduction exists because these B-sector expressions satisfy the remarkable identity

$$B^{\rm boson} ~=~ B^{\rm fermion} ~=~ 4\,q^{1/2}\prod_{n=1}^\infty \frac{(1+q^{3n})^4\,(1-q^{3n})^2}{(1-q^n)^6} ~. \tag{2.8}$$

Note that the right side of this equation is simply the q-expansion of ${\vartheta_2}^2(3\tau)/\eta^6(\tau)$. Therefore, just as the contributions from the A-sector states were isomorphic to those of free worldsheet bosons and fermions (where such fermions are equivalent to internal bosons compactified on circles of radius $R = 1$), we see that the contributions from the B-sector states are also isomorphic to free worldsheet bosons and fermions, but with the fermions on rescaled compactification lattices (or equivalently, with the internal bosons compactified on circles of radius $R = \sqrt{3}$ rather than $R = 1$) [18, 19].

Thus, the appearance of these new B-sector states suggests the existence of additional purely massive BPS states, fully consistent with $c = 6$, but beyond those described in Ref. [11].

3 Massless States and the Internal Projection

The next step is to interpret the partition function (2.7) that we have constructed, and in particular to construct a worldsheet theory for which this function emerges as the partition function. After all, while this partition function has been constructed with a number of properties in mind and on the basis of several remarkable identities, it is *a priori* far from clear that there exists a consistent worldsheet theory underlying it. In this section, we will give evidence that these partition functions are consistent with a theory whose massless states are those of the six-dimensional Green-Schwarz string (as required), and also give evidence that the internal projection is ultimately consistent.

Although we have constructed the partition function (2.7) in order to satisfy certain properties expected of the six-dimensional self-dual string, it remarkably turns out that this partition function is precisely the partition function that was derived several years ago for the so-called $K = 4$ fractional superstring. Detailed discussions concerning these fractional superstrings can be found in the original references [13, 20, 14, 21, 22, 23, 18, 19]. Note that the $K = 4$ fractional superstring was originally constructed as a generalization of the ordinary superstring or heterotic string, but with critical spacetime dimension $D_c = 6$. As such, in its original formulation, it was a theory of gravity and contained a graviton. However, as we shall see, it is possible to modify the original interpretation of these partition functions in such a way as to yield a suitable worldsheet formulation of the self-dual string. Hence, although they share the same partition functions, these new candidate strings are not to be identified with the original fractional superstrings, and knowledge of fractional superstrings will not be needed for what follows.

3.1 The massless sector

We shall concentrate on the left-moving worldsheet modes, since the left- and right-moving modes can ultimately be combined to form the states of the closed string. Before the internal projection, we have seen that we can describe the Fock

space of our six-dimensional string theory in terms of its pre-projection $c = 8$ conformal field theory (CFT), namely a tensor product of four free bosons along with four copies of the $\mathbb{Z}_4$ parafermion theory:

$$\text{original CFT} \;=\; \left\{ \bigotimes_{i=1}^{4} X^i \right\} \;\otimes\; \left\{ \bigotimes_{a=1}^{4} (\mathbb{Z}_4 \text{ PF})^a \right\} . \tag{3.9}$$

This means that every state with spacetime mass M in the Fock space can be written in the form $\phi_h|0\rangle$ where $|0\rangle$ is the vacuum state of the original CFT (3.9), and where ϕ_h is a field in this theory of conformal dimension h. Note that $M^2 = h - c/24 = h - 1/3$.

As indicated in (3.9), the four transverse coordinate bosons X^i carry a spacetime vector index i spanning the $(\mathbf{2},\mathbf{2})$ representation of the $SU(2) \times SU(2)$ massless little group. The zero modes of these bosons are then interpreted as the transverse momenta of the states. By contrast, the fields of each parafermion theory are assigned a spacetime spinor index a transforming in the $2(\mathbf{2},\mathbf{1})$ representation of $SU(2) \times SU(2)$.

With a little knowledge of the $\mathbb{Z}_4$ parafermion conformal field theory, it is easy to construct the lightest states which contribute to the partition function (2.7). As mentioned above, the $\mathbb{Z}_4$ theory is particularly simple to work with because it has $c = 1$, and is equivalent to a free boson ϕ compactified on a circle of radius $R = \sqrt{3/2}$. The primary fields and conformal dimensions corresponding to the $\mathbb{Z}_4$ characters χ_0^0, χ_2^2, χ_0^2, and χ_2^4 are thus respectively identified as

$$\begin{aligned} \mathbf{1} \quad &\Longleftrightarrow \quad h = 0 \ , \\ \exp\left(\pm i\phi/\sqrt{6}\right) \;\equiv\; \sigma_\pm \quad &\Longleftrightarrow \quad h = 1/12 \ , \\ \exp\left(\pm 2i\phi/\sqrt{6}\right) \;\equiv\; \epsilon_\pm \quad &\Longleftrightarrow \quad h = 1/3 \ , \\ \exp\left(\pm 3i\phi/\sqrt{6}\right) \;\equiv\; \psi_\pm \quad &\Longleftrightarrow \quad h = 3/4 \ . \end{aligned} \tag{3.10}$$

Note that the $\sigma_\pm$, $\psi_\pm$, and $\epsilon_\pm$ fields are respectively the spin operator, parafermion current, and energy operator in the $\mathbb{Z}_4$ parafermion theory.

Given this identification of fields, it is straightforward to determine the massless states that contribute to the partition function (2.7). Note that massless states can only contribute to the term $4(\chi_0^0)^3\chi_0^2$ within A^{fermion}, or the term $4(\chi_2^2)^4$ within A^{boson}. The corresponding massless states in the pre-projection conformal field theory (3.9) from A^{fermion} are thus identified as

$$|f\rangle \;=\; (\epsilon_\pm)^a_{-1/3}\, |0\rangle \tag{3.11}$$

where the $\epsilon_\pm$ fields are defined in (3.10) and their modings are given as a subscript. These unusual modings follow from the fractional spin of this parafermionic field, and are just what is required in order to create a massless state. As before, the superscript denotes the spacetime spinor index assigned to each parafermion factor. Thus, these states describe eight spacetime fermionic degrees of freedom transforming as four

Weyl fermions, and fill out the $4(\mathbf{2},\mathbf{1})$ representation of the little group $SU(2) \times SU(2)$.

In a similar fashion, the massless states contributing to A^{boson} are identified as

$$|b\rangle \;=\; (\sigma_\pm)^1_{-1/12}(\sigma_\pm)^2_{-1/12}(\sigma_\pm)^3_{-1/12}(\sigma_\pm)^4_{-1/12}\,|0\rangle. \tag{3.12}$$

To determine the spacetime interpretation of these states, we observe that when acting on $|b\rangle$, the parafermionic $\tilde{\epsilon} \equiv \epsilon_+ + \epsilon_-$ fields remarkably have zero modes which satisfy a Clifford algebra [21]:

$$\{\tilde{\epsilon}^a_0, \tilde{\epsilon}^{a'}_0\}\,|b\rangle \;=\; \delta^{a,a'}\,|b\rangle\ . \tag{3.13}$$

Note that this is a Clifford algebra involving spacetime *spinor* indices, as in the Green-Schwarz string. This implies that the massless $|b\rangle$ states fill out a representation of this algebra. It is easy to see that the spacetime Lorentz properties of such a representation are $8(\mathbf{1},\mathbf{1})\oplus 4(\mathbf{2},\mathbf{1})$, thus describing eight scalar bosons and four Weyl fermions. We suppose that these fermionic states are removed by an analogue of the GSO projection in the Ramond sector of the Neveu-Schwarz-Ramond superstring. This is consistent with the spacetime supersymmetry of the partition function.

Thus, we have identified the massless states in the pre-projection conformal field theory, and find that they fall into massless hypermultiplets of six-dimensional $N = (1,0)$ supersymmetry. Upon combining left- and right-moving sectors in a chiral way, we obtain massless states in the tensor multiplet of $N = (2,0)$ supersymmetry. Note that this description of the self-dual string is a peculiar mixture of the Green-Schwarz and Neveu-Schwarz-Ramond formulations of the ten-dimensional superstring: on the one hand some worldsheet fields carry spacetime spinor quantum numbers, and on the other hand a GSO-like projection removes the tachyonic as well as some of the massless states from the spectrum.

3.2 The internal projection

We now discuss the effects of the "internal projection" which removes states from the spectrum of this theory. Recall that this internal projection is associated with the minus signs appearing within the expressions $A^{\text{boson,fermion}}$ and $B^{\text{boson,fermion}}$ in (2.5) and (2.6). In particular, note that these minus signs precede the characters of sectors which contain only massive states. Thus, this internal projection only affects the *massive* string states, which were the source of Lorentz symmetry-breaking in the usual formulation of the six-dimensional Green-Schwarz string. Moreover, since this internal projection removes only massive states, the description of the massless states given above remains unaffected.

Although this projection may seem similar to the ordinary GSO projection, it has some unusual features. For example, if we arrange the states of the ordinary superstring into towers of conformal descendents of primary states (Verma modules), then the GSO projection in the ordinary superstring either leaves a given tower intact,

or or projects it out entirely. By contrast, in the self-dual string partition function (2.7), the new internal projection which appears projects out only *some* of the states in each individual tower, leaving behind a seemingly random set of states which therefore cannot be interpreted as the complete Fock space of the original underlying worldsheet CFT.

Ordinarily, this would seem to render the spacetime spectrum of this theory inconsistent with any underlying worldsheet-theory interpretation. Remarkably, however, evidence suggests [18, 19] that the residual states which survive the internal projection in each tower precisely recombine to fill out the complete Fock space of a *different* underlying CFT. Thus, whereas the ordinary GSO projection merely removes certain highest-weight sectors of the worldsheet conformal field theory, this new internal projection appears to actually change the underlying conformal field theory itself. In fact, as we have seen above, the effective central charge of the resulting post-projection CFT is smaller than that of the original (parafermionic) CFT. Thus, the internal projection removes exponentially large numbers of states from each mass level of the original Fock space (so as to alter the asymptotic behavior of the degeneracy of states). Indeed, such a drastic projection somewhat resembles the BRST projection which enables unitary minimal models with $c < 1$ to be constructed from free $c = 1$ bosons in the Feigin-Fuchs construction.

In practical terms, the appearance of this internal projection means the following. Although the original worldsheet theory was described by (3.9), we see that the internal projection deforms this theory into a smaller theory with $c = 6$:

$$\text{new CFT} ~=~ \left\{ \bigotimes_{i=1}^{4} X^i \right\} ~~\otimes~~ \{\text{internal } c\text{=}2 \text{ theory}\} ~. \tag{3.14}$$

This not only means that some of the original states $\phi_h|0\rangle$ are projected out of the spectrum, but also that those states which survive can equivalently be written in the form $\phi'_{h'}|0\rangle_{\rm new}$ where $|0\rangle_{\rm new}$ is the new vacuum state of the post-projection conformal field theory (3.14), where $\phi'_{h'}$ is a field of conformal dimension h' in this new CFT, and where the spacetime mass M can be equivalently identified as $M^2 = h' - c'/24 = h' - 1/4$.

Verifying that the internal projection leaves behind a self-consistent CFT is a difficult task, since it requires detailed knowledge of precisely which original parafermionic states $\phi_h|0\rangle$ are projected into or out of the spectrum. This in turn requires an internal-projection operator, constructed out of the parafermionic fields, which would enable us to analyze these projections at the level of individual states. Unfortunately, despite certain clues involving the so-called parafermionic "twist current" [23, 18, 19], no such internal-projection operator has been constructed.

In some sense, then, this worldsheet theory is in a position that can best be appreciated by imagining that the history of the ordinary superstring had been different, and that modular invariance had been discovered *before* the discovery of the GSO projection. Starting with the RNS model, one would have calculated the full

(unprojected) RNS spectrum, and then would have constructed a modular-invariant partition function. One would have observed, in this partition function, a minus sign that signalled a projection on the Fock space of states. This would have seemed mysterious, and it would only later have been discovered that the correct way to implement this projection would be through the G-parity operator. In the present case, we have calculated a unique partition function, and have observed some unexpected minus signs. In our case, however, these signs indicate not an ordinary GSO projection, but rather a mysterious internal projection.

The most compelling evidence to date for the consistency of the internal projection been obtained through a detailed analysis of the partition function. This evidence is examined in detail in Ref. [19], where a full discussion of the known properties (central charge, highest weights, fusion rules, and characters) of the post-projection CFT is given. Non-technical discussions can also be found in Refs. [24, 25]. However, actually constructing a suitable representation for the post-projection conformal field theory in terms of worldsheet fields remains an open question.

3.3 Self-dual strings in lower dimensions

Along with the six-dimensional non-critical self-dual string, it has been pointed out [9] that there also exist four-dimensional $N = 2$ supersymmetric string vacua whose low-energy excitations include light (and tensionless) strings. Therefore, just as with the six-dimensional self-dual string, it may be conjectured that there exists a worldsheet formulation of this lower-dimensional string as well.

One of the advantages of the parafermionic formulation we have proposed above is that it readily generalizes to four and three spacetime dimensions [13, 14]. In particular, the four-dimensional analogue of the above partition function consists of the characters of two transverse coordinate bosons along with those of two $\mathbb{Z}_8$ parafermions, while the three-dimensional analogue consists of the characters of a single transverse coordinate boson along with that of a single $\mathbb{Z}_{16}$ parafermion. In each case, the resulting partition functions are consistent with spacetime supersymmetry, and have the appropriate central charges $c = 3$ and $c = 3/2$ respectively. Just as in the six-dimensional case, these lower-dimensional central charges are obtained in an analogous fashion as the result of an internal projection on the corresponding parafermionic Fock spaces. These lower-dimensional partition functions also contain additional unexpected sectors which are completely analogous to the B-sectors in the six-dimensional case, and which satisfy identities analogous to (2.8). Details can be found in Refs. [18, 19].

Furthermore, note that there also exist $N = (1, 0)$ versions of the six-dimensional self-dual string theories [5, 6, 7, 8]. These may very well be realized as heterotic-like constructions involving the parafermionic worldsheet theories we have proposed here. Indeed, modular-invariant partition functions of this nature have already been constructed [20], and may be interpreted in a similar manner.

4 Discussion

In this note, we have proposed a framework for a worldsheet theory which might be a reasonable candidate for the worldsheet theory of the six-dimensional self-dual string. As far as we are aware, our proposal is presently the only known six-dimensional candidate theory which is spacetime supersymmetric, has central charge $c = 6$, and gives rise to a partition function consistent with (2.1).

Due to their unusual parafermionic formulation, however, we see that the method by which quantum consistency is restored in these worldsheet theories is quite different than the solution proposed in Ref. [12] for the Green-Schwarz string: rather than introduce extra excitation modes (which increase the central charge beyond its desired value), these parafermionic worldsheet theories instead give rise to new sectors with highly unusual properties and to an internal projection which deforms the underlying parafermionic Fock space. Such mysterious sectors and projections may therefore prove to be the key to understanding the worldsheet physics of the conjectured six-dimensional self-dual string.

Of course, we do not understand this theory very well yet. Indeed, as we have discussed above, we only can only represent this theory by starting from a (larger) parafermionic CFT formulation, followed by a highly non-trivial internal projection. Is there a direct formulation of this theory? This remains the major question. Thus, given the present state of knowledge concerning either the self-dual string or the post-projection light-cone gauge worldsheet CFT we are constructing, it is not yet possible to make a more concrete test of this proposal. Such issues await a proper formulation and understanding of the internal projection, and a proper elucidation of the role played by the extra parafermionic B-sectors.

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